Date:

Chapter: Chapter 4:6 --> Inverse Matrices & Systems of

Equations

Objectives: Find the inverse of a 2x2 matrix.

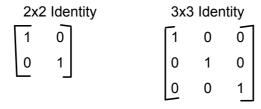Write and solve matrix equations for a system of equations.

Notes:

Maria's Sandwich Shop offers three lunch options as shown at the right. To determine how much each individual item costs, you can solve the following matrix equation in which w represents the cost of a sandwich, s the cost of a side, and d the cost of a drink.

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 4 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} w \\ s \\ d \end{bmatrix} = \begin{bmatrix} 9 \\ 16.50 \\ 30.75 \end{bmatrix}$$

*__Identity Matrix__ (I) = Square matrix, when multiplied by another matrix, = the same matrix; like multiplying by 1.

2x2 Identity

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3x3 Identity

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*__Inverse Matrices__ (A^{-1}) = Two nxn matrices that multiply to be the identity matrix; $A \cdot A^{-1} = A^{-1} \cdot A = I$

--Not all matrices have an inverse!--

Inverse of a 2x2 Matrix

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

*__Matrix Equation__ = Matrix used to solve a system of equations.

*__Variable Matrix__ = Matrix with only the variables of the system of equations.

*__Constant Matrix__ = Matrix with only the constants of the system of equations.

$$\frac{1}{2} 2x = 8 \cdot \frac{1}{2}$$
$$x = 4$$

A . X = B

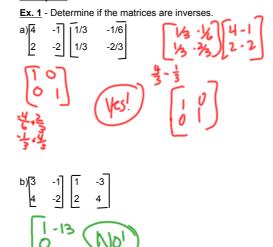-Ex: $\begin{cases} x + 2y = 9 \\ 3x - 6y = 3 \end{cases}$ $\begin{bmatrix} 1 & 2 \\ 3 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

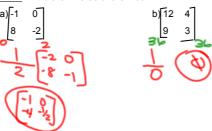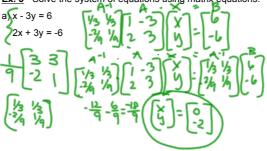Solve: ax = b   AX = B

$$\frac{1}{a} a \cdot x = \frac{1}{a} \cdot b \qquad A^{-1}AX = A^{-1}B$$
$$1x = \frac{1}{a} \cdot b \rightarrow \frac{b}{a} \qquad IX = A^{-1}B$$

Examples:

Ex. 1 - Determine if the matrices are inverses.

a) $\begin{bmatrix} 4 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 1/3 & -1/6 \\ 1/3 & -2/3 \end{bmatrix}$

$\begin{bmatrix} 1/3 \cdot 1/6 \\ 1/3 \cdot 2/3 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & -2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{4}{3} - \frac{1}{3}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(Yes!)

b) $\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$\begin{bmatrix} 1 & -13 \\ 0 & \end{bmatrix}$　(No!)

Ex. 2 - Find the inverse of the matrix.

a) $\begin{bmatrix} -1 & 0 \\ 8 & -2 \end{bmatrix}$

$\frac{1}{2}\begin{bmatrix} -2 & 0 \\ -8 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ -4 & -1/2 \end{bmatrix}$

b) $\begin{bmatrix} 12 & 4 \\ 9 & 3 \end{bmatrix}$

$\frac{1}{0}$　(∅)

Ex. 3 - Solve the system of equations using matrix equations.

a) $\begin{cases} x - 3y = 6 \\ 2x + 3y = -6 \end{cases}$

$\frac{1}{9}\begin{bmatrix} 3 & 3 \\ -2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1/3 & 1/3 \\ -2/9 & 1/9 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

$-\frac{12}{9} - \frac{6}{9} = -\frac{18}{9}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

b) $\begin{cases} -x - 3y = 2 \\ -4x - 5y = 1 \end{cases}$

$\begin{bmatrix} -1 & -3 \\ -4 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\frac{1}{-7}\begin{bmatrix} -5 & 3 \\ 4 & -1 \end{bmatrix}$

$\begin{bmatrix} 5/7 & -3/7 \\ -4/7 & 1/7 \end{bmatrix}$

$\frac{10}{7} - \frac{3}{7} = \frac{7}{7}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Ex. 4

The booster club for North High School plans a picnic. The rental company charges $15 to rent a popcorn machine and $18 to rent a water cooler. The club spends $261 for a total of 15 items. How many of each do they rent?

$\begin{cases} 15p + 18w = 261 \\ p + w = 15 \end{cases}$

Homework:

Average (+10) --> p. 233 (#14-24 Evens, 28-34 Evens)

Advanced (+14) --> p. 233 (#14-24 Evens, 28-34 Evens, 38,

43-45)